SUPERCONDUCTIVITY

AN INTRODUCTION

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Translation by **Timothy Ziman**

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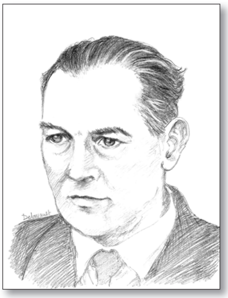
1.2.2 - The magnetic behavior of superconductors

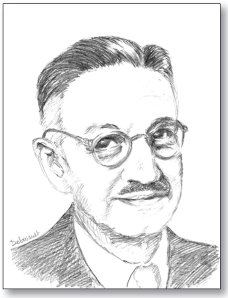
If the drop to zero of the electrical resistivity of superconductors is the most spectacular phenomenon, their response to a magnetic field was just as unexpected and has turned out to be particularly rich in consequences.

The MEISSNER-OCHSENFELD effect



Robert OCHSENFELD

In 1933, in Berlin, Walther MEISSNER and Robert OCHSENFELD showed that magnetic field **B** is "expelled" from superconductors, that is to say that when subjected to an external magnetic field, they divert the field lines so that the magnetic field vanishes inside [2]. The superconducting material behaves as a perfect diamagnet. [3]



Walther MEISSNER

Critical fields and superconductors of types I and II

Very early on, magnetization measurements showed that the superconducting phase existed in a limited range, not only of temperature but also of magnetic field. After much confusion and conflicting experimental results it was finally the theoretical analysis of A. ABRIKOSOV [4] in 1957 that showed that superconductivity can disappear via two distinct scenarios, thus leading to the classification of superconducting materials into those of type I and of type II.

In a superconductor of type I, the superconductivity vanishes abruptly at a critical value H_c of the field. H_c is always small, with $\mu_0 H_c$ no more than 0.1 tesla. Only pure elemental superconductors (with a few exceptions, such as Niobium), are of type I.

In a type II superconductor, there is no discontinuity to be seen, but rather a gradual weakening of the magnetic response starting from a lower critical magnetic field H_{c1}. Complete suppression of superconductivity occurs only when the field reaches an upper critical value H_{c2} which can be very high ($\mu_0 H_{c2}$ may be several tens of, or even a hundred, teslas). Superconducting compounds and alloys are all of type II.

1.2.3 - Critical current

As well as the temperature and magnetic field, a finite density of electrical current also destroys superconductivity when it exceeds some critical value. We shall see

2 W. MEISSNER, R. OCHSENFELD (1933) *Naturwissenschaffen* **21**, 787.

3 W. MEISSNER and R. OCHSENFELD interpreted their result as seeing "a possible analogy to ferromagnetism"; this will be taken up by the LONDON brothers. It is true that W. HEISENBERG had just provided a "microscopic" quantum theory based on interactions between the spins of closely neighbouring electrons.

4 A.A. ABRIKOSOV (1957) *Sov. Phys. JETP* **5**, 1974.

Table 2.1 - Values for the LONDON penetration depths as calculated and measured for a few metals

Element	Al	Sn	Pb	Cd	Nb
Theoretical λ_L [nm]	10	34	37	110	39
Measured λ [nm] extrapolated to 0 K	50	51	39	130	44

2.5.2 - Temperature dependence of the LONDON penetration depth

Experiments show that the LONDON penetration depth $\lambda_L(T)$ increases slowly at low temperatures and diverges approaching the transition temperature T_c (Fig. 2.7). The empirical law quoted most often to represent its behavior is

$$\lambda_L(T) = \lambda_L(0)\left[1-\left(\frac{T}{T_c}\right)^4\right]^{-1/2} \tag{2.31}$$

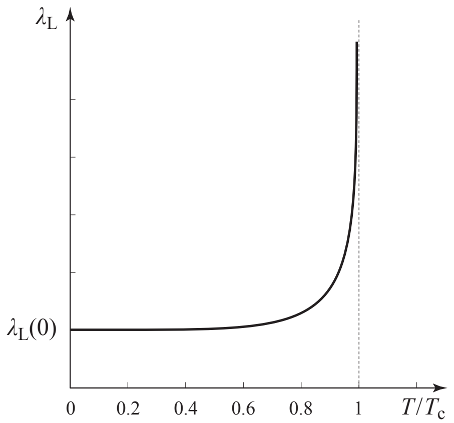
with near T_c

$$\lambda_L(T) \approx (T_c - T)^{-1/2} \tag{2.32}$$

implying, from relations (2.24) and (2.32),

$$n_s(T) \approx (T_c - T) \text{ near } T_c\,. \tag{2.33}$$

This also indicates that the superconductivity disappears with the "conversion" of the superconducting electrons to normal electrons.



The LONDON penetration depth increases slowly at low temperatures and diverges at T_c. This divergence is due to the disappearance of the superconducting electrons.

Figure 2.7 - Thermal dependence of the LONDON penetration depth

2.6 - Applications to superconducting wires

2.6.1 - A wire in magnetic field

The behavior of a cylindrical superconductor of radius R, placed in a magnetic field $\mathbf{B}^0$ parallel to its axis (Fig. 2.8) is not fundamentally different from that of a slab.

This was a distinction that PIPPARD could not have made at the time. With the benefit of hindsight, and in order to clearly separate these two contributions, we will discuss first the case of the pure superconductor, and afterwards the dirty superconductor that PIPPARD had addressed directly with his formula (3.25).

3.2 - Non-locality in pure superconductors

The intrinsic non-locality originates in the fact that superconductivity is carried by COOPER pairs formed by two electrons that can be very far apart (up to several hundreds of nanometers), while intuitively the current density $\mathbf{j}(\mathbf{r})$ may be better identified with the displacement of their centers of gravity. With such a description, we might well question the local form of the proportionality between $\mathbf{j}(\mathbf{r})$ and $\mathbf{A}(\mathbf{r})$ (eq. 2.94) since, if they are far apart, the two electrons of the same COOPER pair can "feel" very different values of the vector potential (Fig. 3.1).

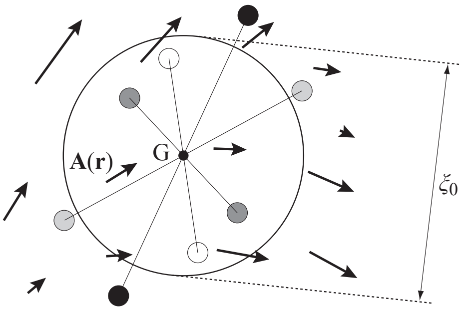


Figure 3.1 - COOPER pairs in a non-uniform vector potential
The figure represents COOPER pairs with the same centrer of gravity in a non-uniform vector potential **A**. As they are separated by an average distance ξ_0 two electrons in the same pair "feel" different values of **A**.

Should we retain LONDON's equation (2.94) by taking for **A** its value at the center of gravity of the COOPER pair, or should we instead use some averaging, taking into account the values of **A** where the electrons actually are? The experimental results show that the second solution should be retained. In the *LONDON gauge*, this leads us to write, by analogy with the anomalous skin effect which has similar origins,

$$\mathbf{j}(\mathbf{r}) = -\frac{n_s e^2}{m} K \iiint \frac{\left[(\mathbf{r}-\mathbf{r}')\cdot\mathbf{A}(\mathbf{r}')\right]}{|\mathbf{r}-\mathbf{r}'|^4}(\mathbf{r}-\mathbf{r}')\, \mathrm{e}^{-\frac{|\mathbf{r}-\mathbf{r}'|}{\xi_0}}\, \mathrm{d}^3 r' \tag{3.2}$$

where ξ_0 is (at 0 K) the average value of the distance between two electrons in the same COOPER pair.

For a more thorough analysis, this expression can be put in the form

$$\mathbf{j}(\mathbf{r}) = -\frac{n_s e^2}{m}\left\{\frac{3}{4\pi\xi_0}\iiint\left[\frac{(\mathbf{r}-\mathbf{r}')\cdot\mathbf{A}(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\,\frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}\right]\left[\frac{1}{|\mathbf{r}-\mathbf{r}'|^2}\,\mathrm{e}^{-\frac{|\mathbf{r}-\mathbf{r}'|}{\xi_0}}\right]\mathrm{d}^3 r'\right\} \tag{3.3}$$

where:

Chapter 4

THERMODYNAMICS OF TYPE I SUPERCONDUCTORS

A decisive step in the history of superconductivity was the recognition that there was a true phase to be reckoned with, and the aim of this chapter is to bring a thermodynamic description of that phase. We will restrict ourselves here to the case of type I superconductors whose superconducting and normal phases are separated in the (H, T) plane by a single line[1] defining the critical field $H_c(T)$ (Fig. 4.1a). Type II superconductors, where a mixed state appears, will be treated in Chapter 6.

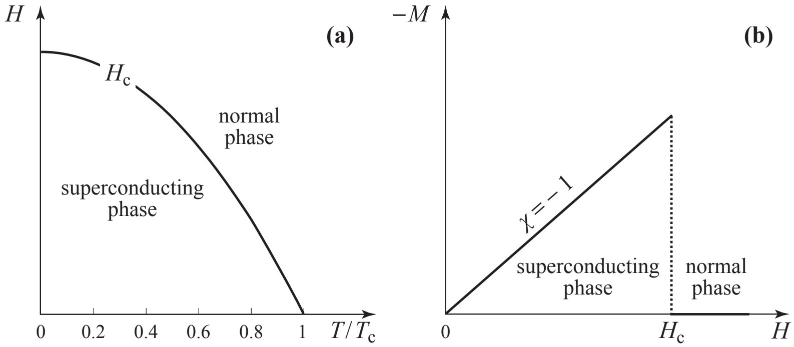


Figure 4.1 - Type I superconductor

(a) The superconducting and normal states are separated by a single line $H_c(T)$. **(b)** In the superconducting state the magnetization **M** is equal and opposite to the field **H**,[2] which makes it a perfect diamagnet. In the normal state, the magnetization vanishes, but in fact the normal state is very weakly diamagnetic. χ is the magnetic susceptibility defined by $\mathbf{M} = \chi\,\mathbf{H}$.

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1 Type I superconductors are, without exception, pure metals of a single element. A few pure metals of a single element, such as niobium and vanadium, are of type II, as are all alloys and compounds.

2 In this book we will encounter **H** and **B** on numerous occasions. The notation in the literature follows (at least) three different conventions. The first is to call **H** "the magnetizing field" and **B** the "magnetic field." The second has **H** the "magnetic field" and **B** the "magnetic induction." The third denotes **B** the "magnetic field" and **H** the "**H** field." We have taken the third option.

the modulus of the total current density somewhere reaches the critical current density of the material.

5.9.2 - Magnetic field applied parallel to the axis of the wire

The LONDON currents are normal to the radial vectors (Fig. 5.13a), and are thus perpendicular to the current carried. The total current density is then

$$j^{\text{tot}} = \sqrt{(j^{\text{trans}})^2 + (j^{\text{screen}})^2}. \tag{5.57}$$

Since each of the densities taking its highest value at the surface,

$$j_{\text{high}}^{\text{trans}} = \frac{I_c(B^0)}{2\pi R\lambda} \quad \text{and} \quad j_{\text{high}}^{\text{screen}} = \frac{B^0}{\mu_0 \lambda} \tag{5.58}$$

the critical intensity $I_c(B^0)$ now depends on B^0. Since the transition towards the normal state begins when j^{tot} reaches j_c, we have

$$j_c^2 = \left(\frac{I_c(B^0)}{2\pi R\lambda}\right)^2 + \left(\frac{B^0}{\mu_0 \lambda}\right)^2 \tag{5.59}$$

which means that, in an axial field $\mathbf{B}^0$, the maximum current intensity that can pass in a type I superconducting wire of radius R is

$$I_c(B^0) = \sqrt{I_c^2(B^0 = 0) - \left(\frac{2\pi R}{\mu_0}\right)^2 (B^0)^2}. \tag{5.60}$$

The dependence of this critical intensity as a function of B^0 is shown in Figure 5.13b. This intensity vanishes for $B^0 = B_c$, beyond which field the wire no longer transports current without energy dissipation.

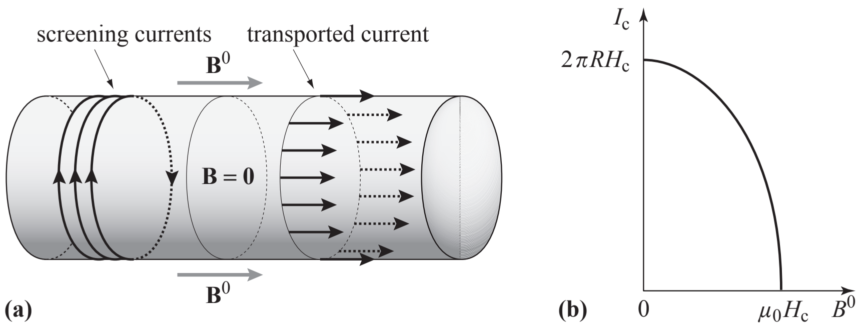

Figure 5.13 - Critical current of a wire in a magnetic field parallel to its axis
(a) Distribution of the transported currents j^{trans} and the screening currents j^{screen} in a superconducting wire carrying a current I^{trans} and subject to an applied field $\mathbf{B}^0$ parallel to the wire. The two distributions of current are orthogonal. **(b)** Variation of the critical current intensity as a function of B^0.

5.9.3 - Magnetic field applied perpendicular to the axis of the wire

As the wire can be considered as a very elongated ellipsoid, any direction perpendicular to the cylinder axis constitutes a principal axis with associated demagnetizing factor N = ½. The screening currents flow around the direction of the wire's axis with density

$$j^{\text{screen}} = j^{\text{screen}}_{\text{high}} \sin\theta \quad \text{with } j^{\text{screen}}_{\text{high}} = \frac{B^0}{\lambda\mu_0(1-\text{N})} = \frac{2\,B^0}{\lambda\mu_0} \tag{5.61}$$

where θ is the angle between $\mathbf{B}^0$ and the radial vector passing through the point considered (Fig. 5.14). The screening current therefore flows in *opposite directions* on the different sides of the wire and the highest screening current densities occur at two positions on the surface directly opposite each other.

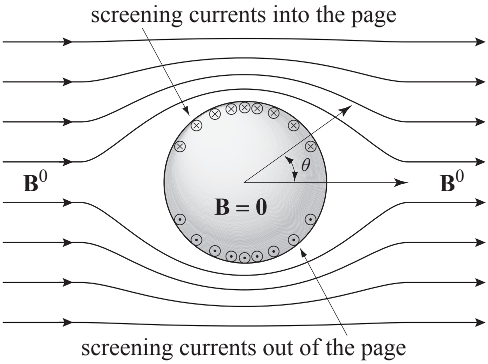

Figure 5.14
Screening currents in a wire placed in a magnetic field perpendicular to its axis (cross-section perpendicular the axis of a cylindrical wire)
The screening currents flow in the direction of the wire's axis, which is a special case of an ellipsoid of demagnetizing factor N = ½. The surface current density varies as sin θ.

The density of transported current is distributed, as always, within the penetration depth, in a *single direction* and it is highest at the surface of the sample where it equals

$$j^{\text{trans}}_{\text{high}} = \frac{I^{\text{trans}}}{2\pi R\lambda}. \tag{5.62}$$

As a consequence, the highest total current density is on the surface of the wire, at the position where the densities of transported and screening currents are simultaneously greatest, and where they flow in the same sense.

In this geometry, the relation between the critical current density and the external field is then

$$\frac{2B^0}{\lambda\mu_0} + \frac{I_c(B^0)}{2\pi R\lambda} = j_c. \tag{5.63}$$

Thus in a transverse field B^0 the maximum current intensity that a type I superconducting wire of radius R can transport without loss is

The normal filament must be replaced by a vortex which is described as a physical "object" of cylindrical symmetry (Fig. 6.8) within which, starting from a central axis, the density of superconducting electrons increases from zero to $n_{s\infty}$ over a characteristic distance ξ and from which the magnetic field decreases from a maximum B_{center} towards zero over the characteristic distance λ. According to the laws of electromagnetism and the second LONDON equation, emergence of the magnetic field **B** requires vortex currents of the LONDON type. These are most commonly considered as screening currents, but here we visualize them more as currents generating an islet of magnetic field within the superconductor.

The name "vortex" is an archaic form of the Latin word "*vertex*" that means "whirlpool."

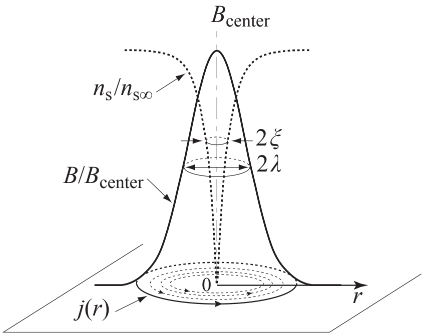

Figure 6.8 - A vortex
Starting from a central axis, the magnetic field B taking a maximum value of B_{center} accompanied with whirling superconducting currents, decreases over the characteristic distance λ while the density of superconducting electrons increases from zero to its bulk value $n_{s\infty}$ over the coherence length ξ. The magnetic field varies little around the vortex center because of the low density of LONDON currents in the core region.

Rigorous determination of the conditions for stability of a vortex requires a precise knowledge of the real profiles of the magnetic field $B(r)$ and the density of superconducting electrons $n_s(r)$. This problem can be treated starting from the GINZBURG-LANDAU equations and leads to a magnetic field which has the form drawn in Figure 6.8. At this stage we make the simple approximation that $n_s(r)$ and $B(r)$ vary exponentially from the central axis [6], each with its characteristic length (ξ and λ respectively) as they would from a planar surface, *i.e.*

$$n_s(r) = n_{s\infty}\left(1 - e^{-\frac{r}{\xi}}\right) \quad ; \quad B(r) = B_{center}\, e^{-\frac{r}{\lambda}}. \tag{6.16}$$

We should not forget that the penetration depth λ is not independent of ξ as the depletion of superconducting carriers reduces the density of LONDON currents that must then spread out more to maintain the magnetic field. In fact this has already been taken into account in going from the LONDON length λ_L to the penetration length λ in the relation (3.6).

6 A more realistic profile has been proposed within the framework of the GINZBURG-LANDAU model.

7.1.1 - Force exerted on a vortex by a transported current

The first situation we consider is when the current density **J**, in the y direction, carries a continuous current across the LONDON regions of the vortices that are present (see Fig. 6.18).

As seen in Figure 7.1, the force acting on each vortex is transverse and directed perpendicularly to the direction of the current.

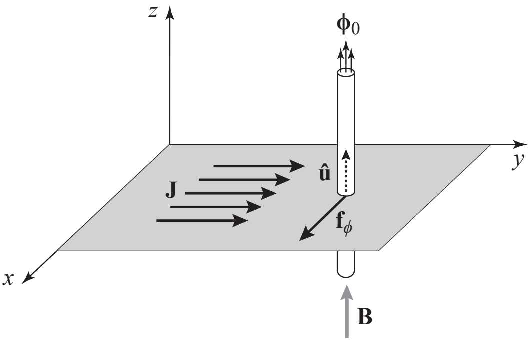

Figure 7.1
Force acting on a vortex in the presence of a uniform current density J
In the presence of a uniform superconducting current density **J**, a vortex is subjected to a transverse force given by the relation (7.1).

7.1.2 - Interaction forces between vortices

Force between two vortices

The second situation is when two vortices V1 and V2 are sufficiently close that each interacts with the vortex current of the other. An element of unit length of the vortex V1, which feels the current density $\mathbf{j}_2$ associated with the vortex V2 (Fig. 7.2), is subject to a repulsive force from that vortex,

$$\mathbf{f}_{1\text{-}2} = \phi_0\, \mathbf{j}_2 \times \hat{\mathbf{u}}_1 \tag{7.2}$$

where $\hat{\mathbf{u}}_1$ is the unit vector along the axis of vortex V1. In return the vortex V2 feels an equal but opposite force from V1.

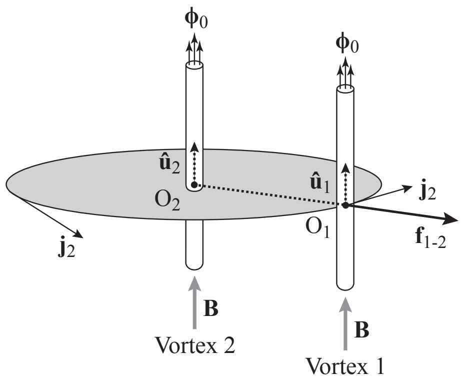

Figure 7.2
Repulsive force between two vortices
As each vortex feels the vortex current of the other, it experiences a repulsive force given by (7.2).

8.6.2 - Nature of the superconducting gap

The superconducting gap is of a nature very different from that of a semiconductor which is due to the periodic potential of the lattice ions. In that case the structure of the energy levels is essentially independent of temperature and the electrons occupy the levels individually with a probability given by the FERMI function. The electron interactions bring only minor corrections.

In superconductors, the gap has its origin in the interactions between electrons. At low temperatures it varies little, but after the quasiparticles occupy the pair states it collapses and falls to zero, which marks the disappearance of superconductivity.

There exists an operational semiconductor representation of the "density of states" of superconductors. In that representation the two bands equivalent to the conduction and valence bands of semiconductors are, for the first, the density of states $D(E)$ of the quasiparticles and for the second $D^*(E) = D(-E)$ (Fig. 8.18). They are separated by 2Δ (the gap in the semiconductor sense).

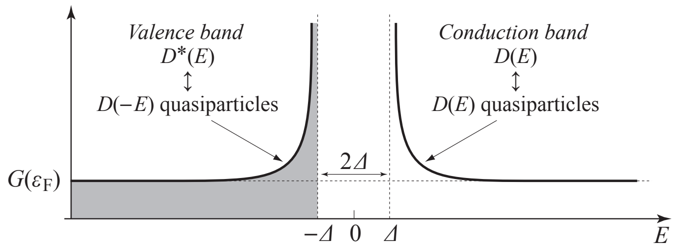

Figure 8.18 - Semiconductor representation

The equivalents of the "conduction band" and the "valence band" are the density of states of quasiparticles and its reflection, respectively. This representation is particularly convenient if we wish to take into account tunneling effects of quasiparticles.

This operational representation is particularly useful to describe the tunneling effects of quasiparticles (to be distinguished from JOSEPHSON effects that are the tunneling of COOPER pairs) and must be restricted to such effects.

8.6.3 - Coherence length[12]

Similarly to the result of section 8.4.5, the average distance between two electrons in a pair is related, by the uncertainty principle, to the spread δk over pair states $|\mathbf{k}_\uparrow, -\mathbf{k}_\downarrow\rangle$ visited by the COOPER pairs. The calculation, like that giving the relation (8.78), leads to the result

$$\xi_{\mathrm{BCS}}(0) = \frac{\hbar v_{\mathrm{F}}}{\pi\Delta(0)} \qquad (8.90)$$

12 We remind the reader: the energy measured from the FERMI level ξ (in regular type) and the coherence length ξ (in italics).

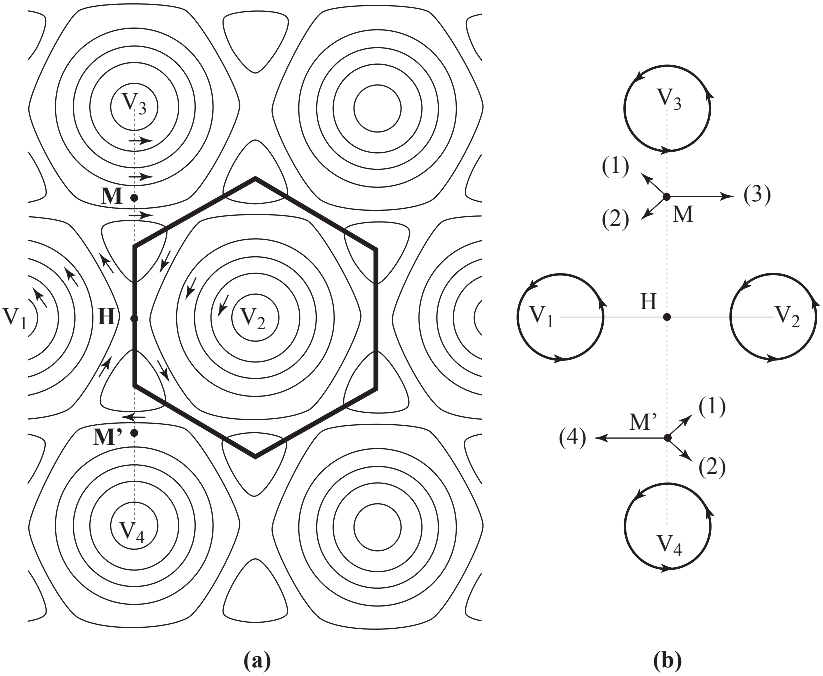

Figure 9.7 - Fluxons in an ABRIKOSOV lattice

(a) Lines of equal density of superconducting electrons and currents in a dense hexagonal ABRIKOSOV lattice [6]. The LONDON currents of two neighboring vortices cancel at the middle of the line separating them. **(b)** The current densities at M and M', imagined as hypothetical superpositions of the currents "associated" with each vortex are normal to the perpendicular bisector of V_1V_2. The circulation of the current density vanishes around the polygon formed by the bisectors between a central vortex and its neighbors (the WIGNER-SEITZ cell). The flux crossing such a cell is equal to the fluxon ϕ_0.

9.5.3 - A confined vortex

In the experiment described in Chapter 6 (section 6.11.5, fig. 6.24), where a vortex is confined to a sample of nanometric scale, the LONDON currents near the surface and the vortex currents turn in opposite directions and are therefore separated by a neutral line where the current density vanishes. Once again, by applying (9.18), the path following this closed line can only be crossed by an integer number of fluxons: one if it is a simple vortex, or several if it is a super-vortex.

The sample itself is subject to a higher flux since we must add the contribution of the decreasing magnetic field in the LONDON zone.

6 W.H. KLEINER, L.M. ROTH & S.H. AUTLER (1964) *Phys. Rev.* **133**, A1226.

Chapter 10

THE JOSEPHSON EFFECT

The JOSEPHSON effect results from the passage of "particles" COOPER pairs, and not of individual electrons, between two superconductors separated by an insulating barrier (SIS), by a normal metal (SNS), by a simple constriction in the superconductor (SCS or "weak link") or by a ferromagnetic layer (SFS). Each of the superconductors, (S_1) or (S_2), hosts a superconducting condensate whose wave function (expression 9.2) possesses its own characteristics: a number of COOPER pairs $n_1(\mathbf{r},t)$ or $n_2(\mathbf{r},t)$ and a phase $\theta_1(\mathbf{r},t)$ or $\theta_2(\mathbf{r},t)$. In the first sections of this chapter we will discuss in detail the "standard" case of the SIS junction, where the COOPER pairs pass from one of the superconductors to the other by tunneling.

The more complex SNS junctions will be introduced in section 10.7. The SFS junction will be the subject of section 10.8. All these junctions are known as "JOSEPHSON junctions."

10.1 - JOSEPHSON equations in an SIS junction

As the thickness of the insulating layer is of order of a nanometer, the wave function of the COOPER pairs of the superconductor (S_1) extends into the superconductor (S_2) and inversely, which leads to a non-zero probability of transfer of COOPER pairs from one to the other by the tunnel effect (Fig. 10.1b).

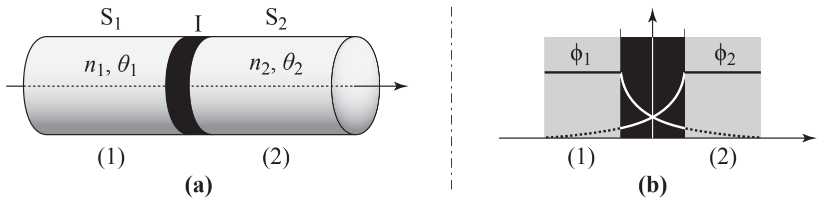

Figure 10.1 - SIS JOSEPHSON Junction
(a) An SIS JOSEPHSON junction is composed of two superconducting blocks (1) and (2) separated by an insulating barrier. Each block is characterized by its number of Cooper pairs n_1 or n_2 and by the phase of its wave function θ_1 or θ_2. **(b)** The overlap of the evanescent parts of the wave functions of the COOPER pairs of each side allows the barrier to be crossed by tunneling.

Chapter 11

SUPERCONDUCTING QUANTUM INTERFERENCE DEVICE "SQUID"

SQUIDs are closed superconducting circuits containing one or more JOSEPHSON junctions. They may be isolated physically and only interact with the external environment by electromagnetic coupling (rf-SQUID) or they may be inserted into part of electrical devices (DC-SQUID). They are at the heart of the most sensitive instruments for measuring magnetic fields. This chapter will introduce us to how they work and describe some of their most common configurations.

11.1 - Nature of the SQUID current

We first consider the simplest SQUID consisting of a superconducting ring interrupted by a single JOSEPHSON junction (Fig. 11.1). A magnetic field flux $\phi = \iint \mathbf{B} \cdot \mathrm{d}\mathbf{S}$ passes through the ring, that carries a current of intensity I. Positive signs for the orientations of $\mathbf{B}$ and I are chosen as indicated in Figure 11.1.

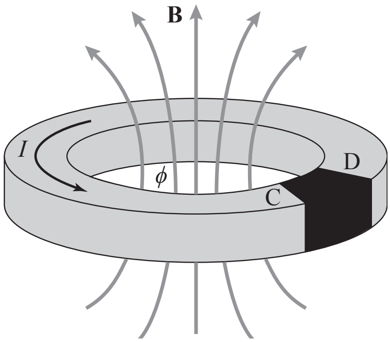

Figure 11.1
Elementary single junction rf-SQUID
The single junction rf-SQUID is a closed superconducting circuit into which is inserted a JOSEPHSON junction between C and D. A current I flows in the circuit through which passes a magnetic field flux ϕ.

In such a circuit there are three terms entering the change in phase of the wave function of the superconducting condensate:

» a phase change due to the circulation of the current density $\mathbf{j}$ along the path DC (the large arc) followed in the positive direction (expression 9.11);

$$(\theta_\mathrm{C} - \theta_\mathrm{D})^{\text{current}} = \frac{m_\mathrm{p}}{n_\mathrm{p} q_\mathrm{p} \hbar} \int_\mathrm{D}^\mathrm{C} \mathbf{j} \cdot \mathrm{d}\mathbf{l}; \tag{11.1}$$

If the pendulum had a long, but not infinitely long, time available it would need to be launched with a minimum angular velocity $\omega^{\text{ext}} \ll 2\Omega$ in order to complete a swing between $t = -\tau/2$ and $t = +\tau/2$. In the language of the junction, an external magnetic field $B^{\text{ext}} \ll B_{\text{cJ}}$ would be necessary to allow a JOSEPHSON vortex to stay within a junction that is long, but not infinitely so.

12.6 - Current transport in a long JOSEPHSON junction

12.6.1 - Long junction carrying a current

At the beginning of section 12.2, we assumed (see relation 12.4) that in zero external field the current density carried was uniformly distributed across the junction. This was, in fact, using the hypothesis of a short junction and we should now return to the problem with the general equations (12.26 to 12.28) or else, if the current is sufficiently small, with their linearized forms (12.30 and 12.31).

In fact, the current distribution will depend on the geometry of the junction. The simplest to analyse is the "in line JOSEPHSON Junction" (Fig. 12.19) where the current circulation in the superconductors electrode creates a magnetic field $\pm B^{\text{el}}$ along the z direction[7] at $y = a/2$ and $y = -a/2$. By application of AMPÈRE's law, and taking the thickness e of the electrode to satisfy $e \ll c$,

$$B^{\text{el}}(+a/2) = \mu_0 \frac{I}{2c} \quad \text{and} \quad B^{\text{el}}(-a/2) = -\mu_0 \frac{I}{2c}. \tag{12.52}$$

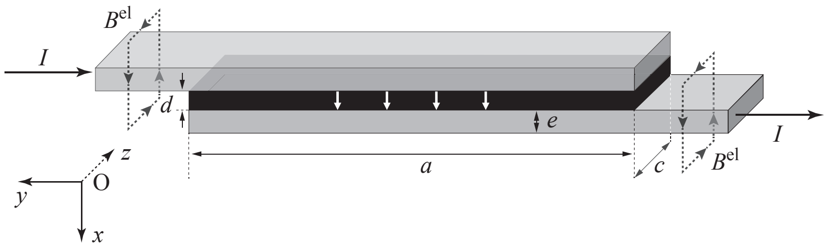

Figure 12.19 - In-line JOSEPHSON junction

The current crosses the isolating layer along the x direction. By application of AMPERE's law along the dotted line ($e \ll c$), the magnetic field created by I circulating in the superconducting electrodes is $\approx \mu_0 I/2c$.

which in the absence of an external field leads to the boundary conditions

$$B_z(+a/2) = \mu_0 \frac{I}{2c} \quad \text{and} \quad B_z(-a/2) = -\mu_0 \frac{I}{2c}. \tag{12.53}$$

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*http://www.wmi.badw.de/teaching/Lecturenotes/AS/AS*_Chapter 2.pdf